

SIT-LP-11/12
November, 2012

Chiral Symmetry and Exact Classical Solution of Nonlinear SUSY Model

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Abstract

The exact classical solution of the equation of the motion for the Nambu-Goldstone fermion of the nonlinear representation of supersymmetry and its physical significance are discussed, which gives a new insight into the chiral symmetry of the standard model for the low energy particle physics.

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Supersymmetry (SUSY) [1], which constitutes a space-time symmetry treating the fermionic and the bosonic degrees of freedom on an equal footing is recognized as the key notion for the unified description of space-time and matter.

The linear representation of SUSY (LSUSY) [2] is realized on the multiplet of fields. Various models based upon $N = 1$ LSUSY have been studied extensively and remarkable phenomenological and field theoretical results have been obtained. The simple version of the minimal supersymmetric standard model (MSSM) with the accessible low TeV SUSY breaking, which may stabilize the mass of light Higgs particle, looks severely constrained by the LHC experiment. The unpleasant point of LSUSY model is that the mechanism and the physical meaning of the spontaneous SUSY breaking, e.g., the origin of D term, is not clear.

While, the nonlinear representation of SUSY (NLSUSY) [3] is realized *geometrically* on specific space-time equipped with the robust spontaneous SUSY breaking encoded in the nature of space-time itself. The action containing higher orders of the fermion field is almost unique up to the higher order derivative terms. Despite the apparent non-renormalizability it may be worth exploring in the NLSUSY framework a new paradigm for the unified theory of nature in terms of the fermion field, which is the long standing challenge. We assume that the ultimate shape of nature is the *four* dimensional (Riemann) manifold possessing the NLSUSY degrees of freedom on tangent space, i.e. tangent space is specified by the Grassmann coordinates $\psi_\alpha (\alpha = 1, 2, 3, 4)$ besides the Minkowski coordinates $x_a (a = 0, 1, 2, 3)$. NLSUSY can be easily fused with the general relativity (GR) principle, which produces the nonlinear supersymmetric general relativity theory (NLSUSYGR) [4, 5] in the form of Einstein-Hilbert (EH) action of the ordinary GR, but with the NLSUSY cosmological term indicating the NLSUSY nature of tangent space. NLSUSYGR *would* break down spontaneously (*Big Decay*) to EH action for ordinary Riemann space-time (graviton) and NLSUSY action for massless Nambu-Goldstone (NG) fermion (called *superon*) called *superon graviton model (SGM)* corresponding to the spontaneous space-time SUSY breaking; [superGL(4,R)/GL(4,R)]. Although NLSUSY is the non-renormalizable highly nonlinear fermionic theory, it is shown by the systematic linearizations of NLSUSYGR in flat space-time that the NLSUSY cosmological term is recasted (equivalent) to the familiar broken LSUSY theory, i.e. the familiar LSUSY theory emerges as the low energy (effective) theory in the true vacuum of NLSUSY theory, where all particles are composites of NG fermion [5, 6]. NLSUSYGR scenario gives new insights into the SUSY effects in cosmology and the low energy particle physics [5, 6].

In this letter we discuss the exact classical solution of the equation of the motion for NLSUSY and show that NLSUSYGR scenario may give a new insight into the chiral symmetry of the SM (MSSM) for the low energy particle physics.

The $N = 1$ NSUSY action L_{NLSUSY} in flat space-time is given by Volkov and Akulov as follows [3];

$$L_{NLSUSY} = -\frac{1}{2\kappa^2}|w| = -\frac{1}{2\kappa^2} \left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \dots \right], \quad (1)$$

where

$$|w| = \det w^a{}_b = \det(\delta^a{}_b + t^a{}_b), \quad t^a{}_b = -\frac{i\kappa^2}{2}(\bar{\psi}\gamma^a\partial_b\psi - \partial_b\bar{\psi}\gamma^a\psi) \quad (2)$$

and κ is an arbitrary constant with the dimension $[length]^{+2}$. L_{NLSUSY} is invariant under $N = 1$ NLSUSY transformation:

$$\delta_\zeta\psi = \frac{1}{\kappa}\zeta - \frac{i}{2}\kappa(\bar{\zeta}\gamma^a\psi - \bar{\psi}\gamma^a\zeta)\partial_a\psi, \quad (3)$$

where ζ is the global spinor parameter. ψ is the coordinate of the coset space $\frac{Super-Poincare}{Poincare}$ and subsequently recasted as the NG fermion in the vacuum of L_{NLSUSY} . κ gives the strength of the coupling of NG fermion to the vacuum.

For simplicity but without the loss of the generality, throughout discussions we adopt the Majorana spinor in the Majorana basis for gamma matrices γ^a , $\gamma_5 (= i\gamma^0\gamma^1\gamma^2\gamma^3)$ and the metric $\eta^{ab} = (1, -1, -1, -1) = \frac{1}{2}\{\gamma^a, \gamma^b\}$, where

$$\gamma^0 = \begin{pmatrix} 0 & -\sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}, \gamma^1 = \begin{pmatrix} 0 & i\sigma^3 \\ i\sigma^3 & 0 \end{pmatrix}, \gamma^2 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \gamma^3 = \begin{pmatrix} 0 & -i\sigma^1 \\ -i\sigma^1 & 0 \end{pmatrix}, \quad (4)$$

$\gamma^{a*} = -\gamma^a$, $\gamma^{0\dagger} = \gamma^0$, $\gamma^{i\dagger} = -\gamma^i$ ($i = 1, 2, 3$), $\gamma_5^\dagger = \gamma_5$, and σ^i are Pauli matrices. The four components Majorana spinor $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ is real in the Majorana basis.

Let us consider the exact solution for ψ of NLSUSY. The variation of (1) with respect to $\bar{\psi}$ gives the following equation of motion for NG fermion ψ ;

$$i\kappa^2\bar{\psi}\psi - (-i\kappa^2)^2 \left(\bar{\psi}\psi T_a{}^a - \gamma_a\partial^b\psi T_b{}^a \right) - \frac{1}{2}(-i\kappa^2)^3 \left[\bar{\psi}\psi \left\{ T_a{}^a T_b{}^b - (T \cdot T)_a{}^a \right\} + 2\gamma_a\partial^b\psi \left\{ (T \cdot T)_b{}^a - T_c{}^c T_b{}^a \right\} \right] = 0, \quad (5)$$

where $T_a{}^b = \bar{\psi}\gamma_a\partial^b\psi = i\kappa^{-2}t_a{}^b$. The highest order term with $\mathcal{O}(T^3)$ is known to vanish identically.

Now we look for the solution of (5). It is not easy to solve (5) in general. However, it is natural to consider the massless case, for ψ is the NG fermion:

$$\bar{\psi}\psi(x) = 0. \quad (6)$$

Substituting (6) into (5) we find

$$\gamma_a \partial^b \psi T_b^a + i\kappa^2 \gamma_a \partial^b \psi (T \cdot T)_b^a = 0. \quad (7)$$

By using the Fiertz rearrangement formula for Majorana spinors we find that (7) is satisfied identically at each order in κ , provided ψ is a *single* chiral state satisfying (6), i.e.,

$$\frac{(1 - \gamma_5)}{2} \psi \equiv \psi_L \text{ or } \frac{(1 + \gamma_5)}{2} \psi \equiv \psi_R, \quad (8)$$

which eliminates terms containing $\bar{\psi}\psi, \partial\bar{\psi}\partial\psi, \dots$ etc. in (7).

For example, by adopting the chiral state ψ_L (or ψ_R) for ψ and by using (6) and the Fiertz formula

$$\gamma_a \partial^b \psi (T \cdot T)_b^a = \gamma_a \partial^b \psi (\bar{\psi} \gamma_b \partial^c \psi) (\bar{\psi} \gamma_c \partial^a \psi) = +\frac{1}{4} \gamma_a \partial^b \psi \sum_{I=1}^{16} (\bar{\psi} \gamma_b \Gamma^I \psi) (\partial^a \bar{\psi} \gamma_c \Gamma_I \partial^c \psi), \quad (9)$$

we can see (9), i.e., the second term of (7) vanishes identically. The situation is similar to the first term of (7). This means that the NG fermion with higher order self-interactions of NLSUSY is the massless chiral fermion which satisfies either of the equation

$$\not{\partial} \psi_L(x) = 0 \text{ or } \not{\partial} \psi_R(x) = 0. \quad (10)$$

These situations are interesting, for the lowest order of T_a^b , i.e., the kinetic term does not constrain the chirality of the (left-right symmetric) solution by itself, however higher order self-interaction terms with NLSUSY structure *as a whole* constrain the chirality of the solution to *one* (left or right) of two chiral eigenstates.

This explains the chiral symmetry for massless fermions in the SM which is a basic assumption within the SM framework, provided NLSUSYGR is the underlying principle. In $N = 2$ minimal SGM scenario ($N = 1$ is unphysical), in fact, SUSYQED (and probably the MSSM as well) emerges in the true vacuum, where all particles are composites of the NG fermion (superon) of NLSUSY, e.g., the fermion λ^I ($I = 1, 2$) for the electron and the neutrino of SUSYQED is composed of the NG fermion ψ^I as follows [6];

$$\lambda^I = \left\{ \psi^I |w| - \frac{i}{2} \kappa^2 \partial_a (\gamma^a \psi^I \bar{\psi}^J \psi^J |w|) \right\}, \quad (11)$$

which means λ^I is the chiral fermion provided ψ^I (superon) is chiral.

The explicit form of the solution of the NG fermion, for example, ψ_L with $\gamma_5\psi_L = -\psi_L$ is expressed in our convention as follows

$$\psi_L = \frac{1}{2} \begin{pmatrix} \psi_1 - i\psi_3 \\ \psi_2 - i\psi_4 \\ \psi_3 + i\psi_1 \\ \psi_4 + i\psi_2 \end{pmatrix}, \quad (12)$$

which is equivalent to a complex two-components Weyl spinor.

In the SGM scenario of NLSUSYGR, the chiral symmetry (and SUSY as well) is broken spontaneously by the vacuum energy density of space-time (NLSUSYGR) recasted into the composite D term and the ordinary symmetry breaking of the SM (MSSM) occurs among corresponding composite fields. The SM (MSSM) may be regarded as the low energy effective theory of NLSUSYGR.

Finally before conclusions, we mention that NLSUSYGR (SGM) scenario constrains the dimensions of space-time to *four*, provided we require $SO(D-1, 1) \cong SL(d, C)$, i.e., $\frac{D(D-1)}{2} = 2(d^2 - 1)$ for the Lorentz invariance of the theory, which holds for only $D = 4, d = 2$ besides the unphysical case $D = 2, d = 1$.

Now we summarize the result as follows. The chiral symmetry for massless quarks and leptons of the SM (MSSM) is understood naturally, if quarks and leptons are (composites of) the NG fermion (superon) degrees of freedom of NLSUSY as demonstrated in the SGM scenario of NLSUSYGR, where higher orders of the stress-energy-momentum tensors of the NG fermion (superon) play an essential role.

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